



Preheating and vacuum metastability in Supersymmetry

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Abstract

The constraints imposed by the requirement that the scalar potential of supersymmetric theories does not have unbounded directions and charge or color breaking minima deeper than the usual electroweak breaking minimum (EWM) are significantly relaxed if one just allows for a metastable EWM but with a sufficiently long lifetime. For this to be acceptable one needs however to explain how the vacuum state reaches this metastable configuration in the first place. We discuss the implications for this issue of the inflaton induced scalar masses, of the supersymmetry breaking effects generated during the preheating stage as well as of the thermal corrections to the scalar potential which appear after reheating. We show that their combined effects may efficiently drive the scalar fields to the origin, allowing them to then evolve naturally towards the EWM.

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One of the peculiar aspects of supersymmetric theories is that they have a very extended scalar sector. In addition to the two complex Higgs doublets of the minimal supersymmetric standard model, the scalar partners of all leptons and quarks are also present, making the structure of the scalar potential of the theory very rich.

The fact that these scalars may carry lepton or baryon number, be electrically charged or even colored makes in principle possible the existence of minima of the scalar potential where the symmetries associated to these charges are spontaneously broken. These so-called charge and color breaking (CCB) minima are of course to be avoided, and the requirement of having the usual electroweak breaking minimum (EWM) deeper than the CCB ones puts important constraints on the parameters, like the soft scalar masses \tilde{m} , the gaugino masses M , the bilinear B and the trilinear A soft breaking terms. Particularly dangerous are the many directions in field space giving vanishing D -terms, *i.e.* the so called D -flat directions, since for them the renormalisable potential can become unbounded from below for large field values due mainly to the effects of the trilinear couplings or to the effects of radiative corrections to the potential. Depending upon the particular directions, the CCB condensate φ may assume values of the same order of the weak scale or (much) larger.

Detailed analysis of the constraints imposed by the requirement of having a well behaved scalar potential have been performed by many authors, resulting in restrictive bounds on the parameter space of supersymmetric theories [1]– [7]. A complete analysis of all the potentially dangerous directions in the field space of the MSSM has been recently carried out in ref. [8], where it was shown that extensive regions of the parameter space (\tilde{m}, M, B, A) become forbidden. The constraints to avoid CCB true minima are sometimes so strong that for instance the whole parameter space is excluded in the particular case in which the dilaton is the source of supersymmetry breaking [9].

It has however been pointed out that imposing that the EWM be the deepest one may actually be exceedingly restrictive, since a deeper minimum or an unbounded direction is harmless as long as the lifetime of the now metastable EWM is longer than the age of the Universe [10,11]. In order to study the metastability of the EWM, one has to consider the quantum tunneling at zero temperature from one vacuum to the deeper one and also take into account the possibility of producing the transition by thermal effects in the

hot early Universe. As a result, it turns out that in many cases the charge and color breaking effects are in practice not dangerous, and hence these kind of considerations can significantly relax the bounds obtained under the requirement of absolute stability of the EWM of the potential [10].

However, in order for these relaxed bounds to be reliable, one has to explain how does the Universe manage to reach the color conserving minimum $\varphi = 0$ in the first place. Indeed, it is possible that the color breaking condensate is left initially far from the origin, *e.g.* at an early epoch near the end of inflation, and may then roll towards some unbounded direction or CCB minimum before reaching the EWM.

The discussion of the conditions under which the Universe may be assumed to populate the EWM at early stages will be the main issue of this paper. In this respect, the determination of the initial conditions may depend crucially on the details of the inflationary epoch and of the subsequent period of inflaton decay and thermalisation.

As we will show, the new idea of the preheating stage [12] produced by the resonant inflaton decay can be helpful to push an initially nonvanishing color breaking condensate to the origin $\varphi = 0$ and hence, when the temperature of the Universe drops down and the electroweak phase transition occurs, to the EWM where it may then remain trapped even if this state is actually metastable, thus relaxing the bounds obtained imposing that the EWM be the deepest one in the parameter space.

There are essentially three main contributions to the scalar potential which can help the vacuum state to naturally evolve towards the origin and avoid becoming trapped in a deeper but far away CCB minimum:

- i)* Scalar mass terms, Δm_H^2 , proportional to the Hubble parameter induced by the inflaton potential energy [13,14].
- ii)* Temperature dependent masses, Δm_T^2 , arising after the thermalization of the Universe.
- iii)* Supersymmetry breaking effects produced by the large scalar condensates generated during the preheating stage [15,16] according to the new theory of reheating [12]. We will indicate the corresponding corrections to the soft breaking masses by Δm_{pr}^2 .

Regarding the first one, it is known that the potential energy responsible for inflation induces a contribution to the soft masses of the form

$$\Delta m_H^2 = c H^2, \quad (1)$$

where H is the Hubble parameter ($H \sim 10^{13}$ GeV during inflation) and c is a constant of order unity. The presence of such a contribution is a signal of supersymmetry breaking during inflation.

For the minimal form of the Kähler potential leading to canonical kinetic terms, c is positive and one may expect that during inflation the color breaking condensate φ is driven exponentially to the origin even if it starts from very large values $\varphi \sim M_P$. In such a case, the considerations discussed in [10,11] may be applied. Nonetheless, the constant c may vanish [17] or even be negative in more general models [14]. Negative values of c along particular flat directions may in fact be required for the Affleck-Dine mechanism of baryogenesis [18] to work, so as to set the scalar fields to large initial values.

We will hence focus hereafter in the more problematic case $c < 0$, and study the evolution of the fields along flat directions which, at zero temperature, present an undesirable minimum φ_0 at large field values (this could typically be the case for an unbounded direction of the renormalisable potential lifted by a non-renormalisable interaction term, see below), and then comment on the ‘non-flat’ case.

The scalar potential for a D -flat direction during inflation and in the presence of a non-renormalisable superpotential of the type $W_{NR} = (\lambda/nM^{n-3})\varphi^n$ can be represented as follows [14]:

$$V(\varphi) = c H^2 |\varphi|^2 + \lambda^2 \frac{|\varphi|^{2n-2}}{M^{2n-6}}, \quad (2)$$

where M is some large mass scale such as the GUT or Planck mass and n is some integer which may take values larger than 3. This leads to symmetry breaking with $\langle \varphi \rangle \sim (HM^{n-3}/\lambda)^{1/n-2}$.

In the old theory of reheating [19], after the inflationary stage the inflaton field ϕ starts oscillating around the minimum of its potential and the Universe soon becomes matter-dominated (assuming a quadratic inflaton potential). In such a stage, $H \propto a^{-3/2}$, $a(t)$ being the expansion scale factor, and the minimum $\langle \varphi(t) \rangle \propto H^{1/n-2}$ decreases accordingly. At the time $t_d \sim \Gamma_\phi^{-1}$, the inflaton decays with a decay width Γ_ϕ and the inflaton energy is released under the form of light relativistic particles which thermalize and reheat the Universe up to a temperature $T_R \simeq 10^{-1} \sqrt{\Gamma_\phi M_P}$. At this epoch, if

the renormalisable terms of the superpotential can still be neglected, the flat direction minimum will be at

$$\langle\varphi\rangle_R \simeq \varphi_d \equiv \left(\frac{\Gamma_\phi M^{n-3}}{\lambda}\right)^{\frac{1}{n-2}}, \quad (3)$$

which for $n = 4$ and $M = M_P$ is of the order of $10 T_R/\sqrt{\lambda}$. Note that, for $c < 0$, the effect of the inflaton induced mass is always to shift the minimum of the potential to values larger than the zero temperature minimum φ_0 , and hence for the previous reasoning to apply the value of φ_d obtained above should be larger than φ_0 . This may not be the case if the renormalisable terms are actually non negligible, in which case one may just assume that $\langle\varphi\rangle_R \simeq \varphi_0$ at the reheating time.

After reheating, the negative correction Δm_H^2 disappears because the inflaton energy has been reduced to zero and the effective potential $V(\varphi)$ consists now of the zero temperature piece and, for not too large values of the field, the thermal effects may provide a new important contribution. Indeed, the fields directly interacting with φ acquire masses $\simeq g\langle\varphi\rangle$, where g is their coupling to the flat direction. Hence, after reheating they may become excited by thermal effects and induce a finite temperature correction of order $g^2 T^2 \varphi^2$ to the effective potential as long as $g\langle\varphi\rangle_R \lesssim T_R$. If the renormalisable terms are small, so that $\langle\varphi\rangle_R \simeq \varphi_d > \varphi_0$, this condition translates into

$$g \lesssim 0.1 \left(\frac{\Gamma_\phi}{M}\right)^{\frac{n-4}{2n-4}} \sqrt{\frac{M_P}{M}} \lambda^{\frac{1}{n-2}}. \quad (4)$$

As an example, this implies that g is to be smaller than $\sim 0.1\sqrt{\lambda M_P/M}$ for $n = 4$, which is not difficult to satisfy. However, for $n \gg 4$ the vacuum expectation value (VEV) $\langle\varphi\rangle_R$ turns out to be quite large ($\sim M$) and one needs then $g \lesssim T_R/M$, a requirement which could be very stringent. If the condition in Eq. (4) is satisfied so that the flat direction soft breaking mass receives a temperature dependent contribution $\Delta m_T^2 \sim g^2 T^2$ due to the effect of the light particles coupled to φ , this new contribution can help to drive the field to the origin. Indeed, if just after reheating this thermal correction dominates the scalar potential for $\varphi \lesssim \langle\varphi\rangle_R$, it may completely drag the CCB condensate to the origin. The detailed conditions under which this happens however depend crucially on the actual value of g , which cannot be too small for Δm_T^2 to be sizeable, and on the strength of the

renormalisable terms in the potential which could compensate the thermal effects and push the field away from the origin.

The same problem is present if the renormalisable terms are sizeable at reheating and make the CCB φ_0 much larger than both φ_d and T_R/g . Again, the time-dependent minimum relaxes towards φ_0 to remain trapped there since there are no significant temperature induced corrections. This is unacceptable and the parameter space of the theory leading to this kind of situation should be eliminated. In this case, only if T_R is large enough (its lower bound depending upon the different situations) it may happen that the thermal effects push the fields towards the origin and allow them to evolve later towards the EWM. On the other hand, the CCB minima whose present VEV's are of the order of the weak scale (such as those along the D -flat direction $\varphi = H_2^0 = \tilde{t}_L = \tilde{t}_R$ or along the direction $\varphi = \tilde{t}_R$ with negative soft breaking mass \tilde{m}_R^2 [11,20]) are very unlikely to be ever populated in the early Universe. Indeed, it is quite reasonable to expect the reheating temperature after inflation to be much larger than the weak scale, making thermal corrections very efficient in driving the color breaking condensate to the origin. Hence, in this case the regions of parameter space for which the lifetime of the EWM is sufficiently large can be considered as cosmologically acceptable and do not pose any problems to the consistency of the theory.

In the rest of the paper we will therefore concern ourselves with the case of CCB minima whose present VEV's are (much) larger than the weak scale and investigate how a preheating stage after inflation may modify the picture arising from the old theory of reheating that we sketched above. Indeed, it has been recently pointed out that the inflaton may decay explosively just at the end of inflation through the phenomenon of parametric resonance [12], leading to a situation quite different from the one predicted in the old theory of reheating which could also be helpful in driving the scalar fields towards the origin.

According to this new scenario, a significant fraction of the inflaton energy is released in the form of bosonic inflaton decay products, whose occupation number is extremely large, and may have energies much smaller than the temperature that would have been obtained by an instantaneous conversion of the inflaton energy density into radiation. Since it requires several scattering times for the low-energy decay products to form a

thermal distribution, it is rather reasonable to consider the period in which most of the energy density of the Universe was in the form of the nonthermal quanta produced by inflaton decay as a separate cosmological era, dubbed as preheating to distinguish it from the subsequent stages of particle decay and thermalization which can be described by the techniques developed in [19]. Several aspects of the theory of explosive reheating have been studied in the case of slow-roll inflation [21] and first-order inflation [22]. One of the most peculiar aspects of the stage of preheating is the possibility of nonthermal phase transitions with symmetry restoration [23–25] driven by extremely large quantum corrections induced by particles generated during the stage of preheating.

The key observation is that fluctuations of scalar fields produced at preheating may be so large that they can break supersymmetry much strongly than inflation itself [15,16]. This may happen since parametric resonance is a phenomenon characteristic of bosonic particles and the resonant decay into fermions is inefficient because of Pauli’s exclusion principle. Therefore, during the preheating stage the Universe is populated only by a huge number of bosons and the occupation numbers of bosons and fermions of the same supermultiplet coupled to the inflaton are unbalanced. Supersymmetric cancellations between diagrams involving bosons and fermions are no longer operative at this stage and large loop corrections appear. The preheating stage is then intimately associated to strong supersymmetric breaking and large fluctuations may lead to symmetry restoration along flat directions of the effective potential even in the theories where the usual high temperature corrections are exponentially suppressed. Hence, the curvature along D -flat directions during the preheating may be much larger than the inflaton induced effective mass. This may render the details of the effective potential along D -flat directions during inflation almost irrelevant as far as the initial conditions of the condensates along these directions is concerned, and may have a profound impact on the fate of CCB minima.

Let us take as an illustrative case the one usually considered in which the inflaton decays into a field χ , and assume that χ is coupled to the flat direction φ , with the potential

$$V = \frac{M_\phi^2}{2}\phi^2 + g_\phi^2\phi^2|\chi|^2 + g_\varphi^2|\varphi|^2|\chi|^2 + g_\chi^2|\chi|^4, \quad (5)$$

where $M_\phi \sim 10^{13}$ GeV in order for the density perturbations generated during the infla-

tionary era to be consistent with COBE data [26] and where the last term could be for instance a D -term self-coupling of χ if this field is not a gauge singlet.

Inflation occurs during the slow rolling of the inflaton field until it reaches a value $\phi_0 \sim M_P$. Then it starts oscillating with an initial amplitude ϕ_0 and a significant fraction of the initial energy density $\rho_\phi \sim M_\phi^2 \phi_0^2$ is transferred to bosonic χ -quanta in the regime of parametric resonance. Let us postpone the discussion of the conditions under which parametric resonance occurs and go directly to the main observation of this paper.

At the end of the broad resonance regime the inflaton field drops down to $\phi_e \sim 10^{-2} M_P$ and the Universe is filled up with χ -bosons with a typical energy $E_\chi \sim 0.2 \sqrt{g_\phi M_\phi M_P}$ and a large occupation number $n_\chi/E_\chi^3 \sim 1/g_\phi^2$. The amplitude of the field fluctuations produced at this stage is very large [12,23]

$$\langle \chi^2 \rangle \sim 5 \times 10^{-2} g_\phi^{-1} M_\phi M_P. \quad (6)$$

It is exactly the incredibly large value¹ of $\langle \chi^2 \rangle$ that leads to nonthermal symmetry restoration during the preheating stage, drives strong supersymmetry breaking and is responsible for the additional contribution to the effective mass along the D -flat direction

$$\Delta m_{\text{pr}}^2 \sim g_\phi^2 \langle \chi^2 \rangle \sim 10^{-1} \frac{g_\phi^2}{g_\phi} M_\phi M_P. \quad (7)$$

The curvature of the effective potential along the D -flat direction becomes large and positive and the symmetry is restored if $\Delta m_{\text{pr}}^2 > \delta |c| H^2$, where $\delta \sim 10^{-1}$ parametrizes the fraction of the energy density still stored in the inflaton field after the end of the preheating stage.

Since at the end of preheating $H \sim 10^{-2} M_\phi$, this happens if

$$g_\phi^2 \gtrsim 10^{-9} g_\phi. \quad (8)$$

Let us note that the scalar background corrections to the masses in Eq. (7) are a consequence of forward scattering processes which do not alter the distribution function

¹If the energy of the inflaton field after preheating were instantaneously thermalized, a much smaller value of $\langle \chi^2 \rangle$ would have been obtained, $\langle \chi^2 \rangle \sim 10^{-4} M_\phi M_P$.

of the particles traversing the χ background, but simply modify their dispersion relation. Since the forward scattering rate is usually larger than the large-angle scattering rate responsible for establishing the thermal distribution, this contribution can be present even before the initial nonequilibrium distribution function of the χ particles relaxes to its thermal value. Actually, in realistic models thermalization typically takes a lot of time and the value of $\langle \chi^2 \rangle$ after the Universe reheats is much smaller than (6). Notice also that Eq. (8) is approximately the same condition which enables the interaction rate of the forward scatterings to be large enough, i.e. $\Gamma_{fs} \simeq \sqrt{\Delta m_{pr}^2} > H$, for producing the correction to the soft breaking masses and lifting the curvature of the D -flat direction.

Hence, we have shown that if the Universe spent an intermediate stage of preheating, it is possible that the initially nonvanishing color breaking condensate was dragged to the origin as a consequence of strong supersymmetry breaking and nonthermal symmetry restoration. If so, even though a deeper CCB minimum may be present in our (almost) supersymmetric world, the considerations of metastability of the EWM mentioned at the beginning may safely be applied in order to explore the allowed parameter space of the theory. All these considerations apply provided the condition (8) is fulfilled so that even a negative inflaton induced mass can be compensated by the preheating effects. To get the feeling of the restrictions this condition poses, let consider the D -flat direction $\tilde{u}_R^1 = \tilde{s}_R^2 = \tilde{b}_R^3 \equiv \varphi$ discussed in refs. [6,10]. If χ is not one of the fields labelling the flat direction, the coupling g_φ should be identified with a Yukawa coupling. If the inflaton couples to \tilde{b}_L or H_1^0 then $g_\varphi = h_b$ and the inequality (8) is easily fulfilled. A more concrete example may be provided in the case in which chaotic inflation is driven by a right-handed sneutrino $\tilde{\nu}^c$ [27]. The latter may decay by parametric resonance into sleptons and Higgses through the coupling in the superpotential $\delta W = h_\nu \nu^c L H_2$. The coupling g_φ is then identified with the Yukawa coupling of the u -quark and condition (8) is satisfied if $g_\varphi \equiv h_\nu \lesssim 10^{-1}$. These simple estimates indicate that, even though the validity of the results depends upon the details of the theory, the mechanism of nonthermal symmetry restoration along CCB D -flat direction may offer a concrete explanation of why does the Universe manage to sit on the color conserving minimum $\varphi = 0$ at early stages.

Once the condensate is driven to the origin, it is likely to stay there even at later epochs. Indeed, after the end of the broad resonance the Universe is radiation dominated

and the value of $\langle \chi^2 \rangle$ decreases approximately as $a^{-2} \sim t^{-1}$. As a result, Δm_{pr}^2 decreases as t^{-1} , while H^2 scales like t^{-2} . Hence the condition $\Delta m_{\text{pr}}^2 > \Delta m_H^2$ continues to be satisfied after the end of the broad resonance stage and the condensate remains trapped at the origin. This remains true even if there is some intermediate stage when the Universe suffers a matter dominated period, e.g. if the residual energy density stored in the form of inflaton oscillations, which at the end of the broad resonant regime was reduced to a fraction δ of the total energy density but however decreases more slowly than the radiation, starts dominating again the energy density of the Universe.

When finally thermalization of the χ background occurs at the time τ , the Universe is reheated to a temperature $T(\tau) \simeq T_R \sqrt{\tau_0/\tau}$ [15], where $T_R \simeq 10^{-2} \sqrt{M_\phi M_P}$ is the temperature which the system would have reached if thermalization occurred instantaneously after the preheating stage and $\tau_0 \sim 10^8/M_P$ is the time at the end of the broad resonance regime². At this moment the D -flat direction is lifted by the term $\sim T^2(\tau)\varphi^2$ arising from the thermal effects associated to the light particles coupled to φ (remember that now $\langle \varphi \rangle \simeq 0$). Since $T^2(\tau) \gg H^2(\tau)$, we may conclude that the CCB condensate will remain trapped near the origin by thermal effects and relax towards the EWM at later stages, unless the coupling g_φ is too small to fulfill Eq. (8).

We now further check the applicability of our predictions and discuss in more detail the requirements for having parametric resonance. For the resonance to be ‘broad’, the coupling of the inflaton to χ should not be too small, $g_\phi \phi_e > M_\phi$, where $\phi_e \sim 10^{-2} M_P$ at the end of preheating. This leads to $g_\phi > 10^{-4}$. Notice that the flatness of the inflaton potential during inflation is preserved for such large values of couplings g_ϕ by supersymmetric cancellations [16]. The contribution $g_\varphi \langle \varphi \rangle \sim g_\varphi (H M^{n-3}/\lambda)^{1/n-2}$ to the effective mass of the field χ induced by the condensate $\langle \varphi \rangle$ at the end of the preheating stage should be smaller than the typical energy of the decay products E_χ . For $n = 4$ and $M = M_P$, this translates into the bound $g_\varphi^2 \lesssim g_\phi$.

²It is worthwhile mentioning that particles popping out from the thermalization processes are not guaranteed to be in thermal equilibrium if the reheating temperature is somewhat higher than 10^{14} GeV [28].

Interactions which reduce the number of χ quanta produced during the inflaton decay may remove the decay products of the inflaton and take the system away from the resonance shell, thus stopping the parametric resonance stage (self-interactions of the χ -field instead just keep particles inside the resonance shell). This can be avoided in two different ways. Either the scatterings are suppressed by kinematical reasons, namely if the nonthermal plasma mass of the final states is larger than the typical energy of the χ 's. Otherwise, and more realistically, scatterings occur, but are slow enough that they do not terminate the resonance. This may happen if the interaction rate $\Gamma \sim n_\chi \sigma$, where $\sigma \propto \alpha^2/E_\chi^2$ denotes the scattering cross section, is smaller than the typical frequency of oscillations at the end of the preheating stage $\sim g_\phi \langle \chi^2 \rangle^{1/2}$. This translates into the bound $\alpha \lesssim 10 g_\phi$.

If χ particles self-interact strongly, e.g. if the χ -field is not a gauge singlet and g_χ is a D -term self-coupling larger than g_ϕ , some particular care is needed [25,29]. The resonance stops when the value of the plasma mass of the χ -field induced by nonthermal effects, $m_\chi^2 \sim g_\chi^2 \langle \chi^2 \rangle$, becomes of the order of $\sqrt{g_\chi n_\chi E_\chi}$, i.e. when the fluctuation $\langle \chi^2 \rangle$ becomes $\sim 5 \times 10^{-2} g_\chi^{-1} M_\phi M_P$ and $n_\chi/E_\chi^3 \sim g_\chi^{-2}$. In such a case, all the conclusions drawn so far remain qualitatively unaltered, even though our picture changes from the quantitative point of view. For instance, nonthermal symmetry restoration along the D -flat direction now occurs if $g_\phi^2 > 10^{-8} \delta g_\chi$ and the resonance is not stopped by production of different quanta if $g_\phi > 10^{-2} g_\chi^3$, where we have assumed $\alpha \sim g_\chi^2$. Thermalization is expected to occur earlier than in the weak coupling limit and the thermalization temperature $T(\tau)$ is consequently higher.

The discussion above makes clear that the validity of the scenario discussed is based on several assumptions about the structure of the theory and on relations between various coupling constants. These assumptions should be explicitly checked when dealing with a completely realistic model of inflation. It is anyhow encouraging that the recent progress in the theory of reheating may help in removing one of the dangers present in any supersymmetric theory, namely the presence of unwanted CCB minima which make the EWM metastable. We have shown that the combined effects of the strong supersymmetry breaking during the preheating stage and the thermal corrections appearing after the reheating may provide a concrete explanation of why the origin of the field space was

populated at early times, allowing the condensate to then evolve naturally towards the metastable EWM at temperatures around the weak scale. Since even in the presence of a deeper CCB minimum the lifetime of the EWM is often longer than the age of the Universe, in these cases the charge and color breaking effects may actually pose no problems.

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